TONBRIDGE SCHOOL

Sample Paper for Entrance into Year 12

Further Mathematics

Time allowed:

30 minutes

Name .....

School .....

Total marks: 30

Instructions:

Answer all questions.

Show all your method in the space provided. You are allowed to use a calculator in this exam. A list of useful formulae will be found on page 3.

Formulae sheet – Higher Tier

Arithmetic series

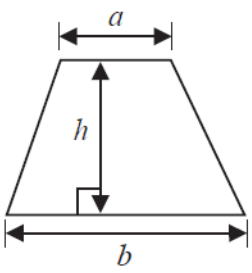
Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

The quadratic equation

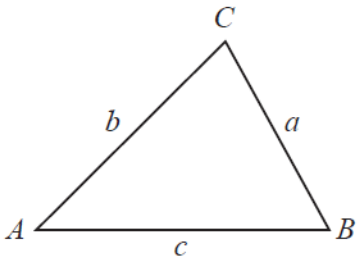
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a+b)h$



Trigonometry



In any triangle ABC

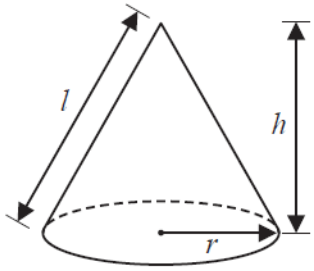
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

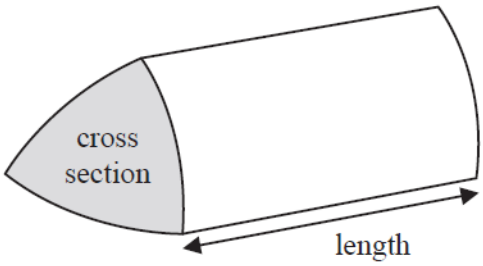
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

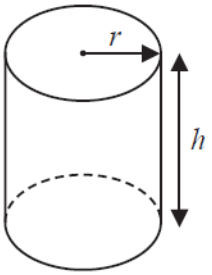
Curved surface area of cone $= \pi r l$



Volume of prism
= area of cross section × length

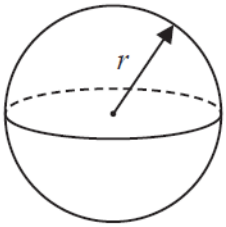


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi r h$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



1 Given that $0 < a < 1$ and $(x + a)(x + 2a)(x + 3b) = x^3 + 6x^2 + \frac{29}{4}x + c$ for all real numbers x, find c.

[6]

2 I have a biased coin and a fair die.

I flip the coin twice and roll the die twice.

The probability that I get one head and one tail with the coin and a total score of 8 on the die is the same as the probability that I flip two tails with the coin and a total score of 10 on the die.

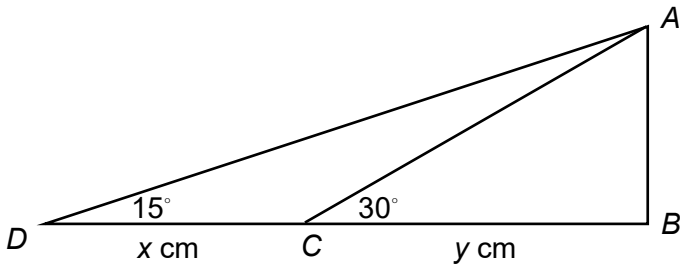
Calculate the probability that, when I flip the coin once, I get a head.

[6]

3 **For this question, you must show all your method and you must not use a calculator.**

The following diagram shows triangles *ABC* and *ABD*.

$A\hat{D}B = 15^\circ, A\hat{C}B = 30^\circ, A\hat{B}C = 90^\circ, CD = x$ cm, $BC = y$ cm.



Find, in exact form, the irrational number *k* such that $y = kx$.

Here is a table of trigonometric values you may use.

Angle, θ°	$\sin\theta^\circ$	$\cos\theta^\circ$	$\tan\theta^\circ$
15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$

[6]

4 Line L_1 passes through the origin with positive gradient.

Line L_2 passes through the point (10,0) and is perpendicular to L_1.

The triangle bounded by the x-axis, the line L_1 and the line L_2 has area 15 square units.

Find the possible gradients of the line L_1.

[6]

5 $n! = n \times (n-1) \times (n-2) \times ... \times 3 \times 2 \times 1$

$N^2 p$ is the difference between the smallest cube number of which 6! is a factor and the smallest square number of which 7! is a factor.

N is an integer and p is a prime.

Find N and p.

[6]