



TONBRIDGE SCHOOL

Scholarship Examination Sample Paper

MATHEMATICS II

Time allowed: 1 hour 30 minutes

There are seven questions in this paper.
Answer as many questions as you can.
All the questions carry equal marks.
You may attempt the questions in any order.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

There is space for extra working at the end of the paper.
If you use it, you should indicate the question number clearly.

- 1 A ball of mass m kg rolls along smooth horizontal table, with speed u ms^{-1} , and collides with an identical ball. The coefficient of restitution e is given by the formula

$$e = \frac{1}{u} \sqrt{\frac{8E}{m}} - 1$$

where E is the kinetic energy, measured in Joules, of the second ball following the collision.

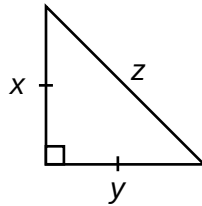
- a Find e when $u = 3.8$, $E = 12.3$, $m = 1.2$.
- b Find u when $e = 0.71$, $E = 15.8$, $m = 5.2$.
- c Find m when $e = 0.59$, $E = 27.3$, $u = 9.4$.
- d Given that $0 \leq e \leq 1$, find the maximum value of E in terms of u and m .

TURN OVER

2 $xy = a$, $xz = 2b$, $yz = 3c$ where a , b , and c are positive numbers.

a Find an equation for each of x , y , and z in terms of a , b , and c .

You are now given that x , y , and z represent the lengths of the sides of an isosceles right-angle triangle as shown in the figure below.

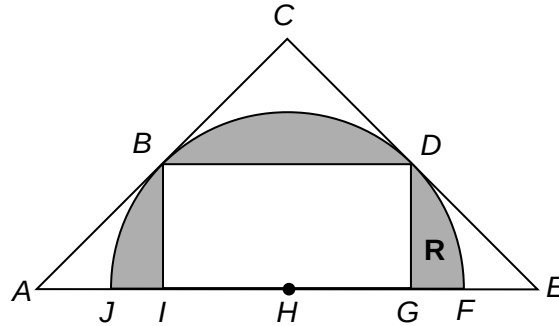


b If $a = \frac{1}{2}$, show that $b = \frac{\sqrt{2}}{m}$ and $c = \frac{\sqrt{2}}{n}$ where m and n are whole numbers that you should find.

TURN OVER

TURN OVER

- 3 The figure below, which is symmetrical about the line CH , shows a logo that a company intends to print in different sizes. The logo is made up of a rectangle $BDGI$, a semicircle, centre H , and a triangle ACE . All three shapes meet at B and D . You may use the fact that the points H, B, C , and D form a square. Points G and I lie on the diameter FJ of the semicircle. Points F, G, H, I , and J lie on the line AE .



- a In a small version of the logo, the radius HF of the semicircle has length 3cm.
 - i Find the area of the region labelled **R** bounded by the lines DG , FG , and the arc DF .
 - ii Find the perimeter of the region **R**.
- b In a larger version of the logo, the unshaded region has total area 200cm^2 . Find the length HF .

TURN OVER

TURN OVER

- 4 The surface area, A , and volume, V , of a regular dodecahedron can be given by the formulae

$$A = \frac{15\phi}{\sqrt{3-\phi}} a^2, \quad V = \frac{5\phi^3}{6-2\phi} a^3$$

where a is the length of one of its edges, and ϕ is the constant known as the golden ratio.

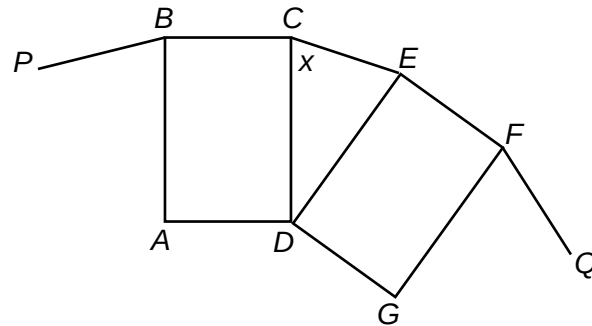
You may use the fact that $\phi = 1.61803$ to 5 decimal places.

- a Find the volume of a regular dodecahedron with surface area 100cm^2 .
- b Find the surface area of a regular dodecahedron with volume 100cm^3 .
- c Find the value of a if the numerical values of A and V are equal.
- d Use algebra to show that $\frac{A^3}{V^2} = k \frac{\sqrt{3-\phi}}{\phi^3}$ where k is a whole number which you should find. You should try to avoid using the numerical value for ϕ that you have been given.

TURN OVER

TURN OVER

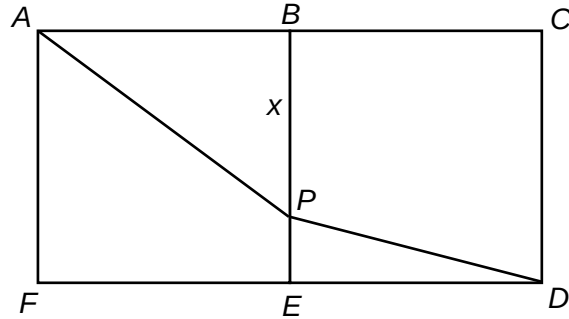
- 5 The diagram shows two rectangles, $ABCD$ and $DEFG$, which each share a side with a regular n -sided polygon, five of whose sides are $PB = BC = CE = EF = FQ$. Angle $\hat{DCE} = x$.



- If $n = 20$, find x .
- If $ABCD$ and $DEFG$ are squares, find n .
- Is it possible for x to be 40° ? Explain your answer carefully.
- When $x = 89.5^\circ$, the lengths AD and DG form adjacent sides of a regular polygon with m sides. Find m .

TURN OVER

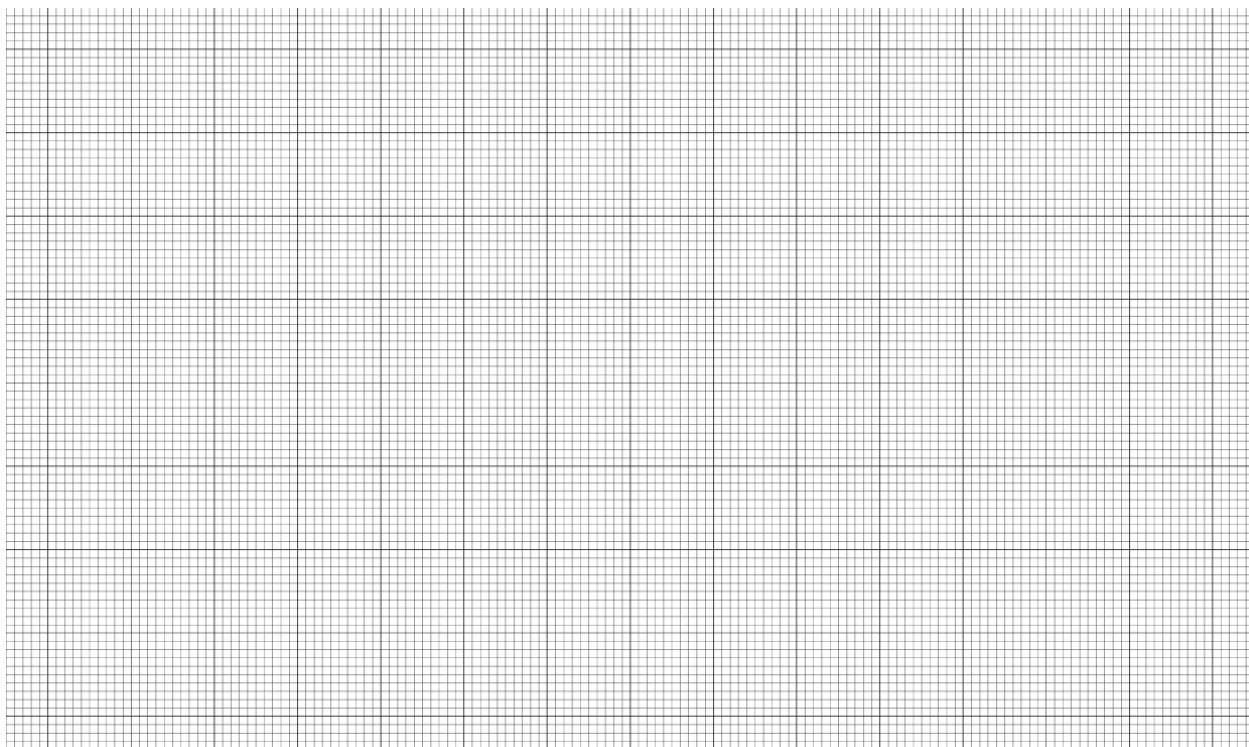
- 6 Old McDonald has a farm consisting of two adjacent square plots of land, $ABEF$ and $BCDE$, of side length 5km. His tractor breaks down at A and he needs to walk back to his house at D . He walks through each square plot in a straight line meeting the line BE at P such that $BP = x$ km as shown in the figure below.



He can walk through plot $ABEF$ at 4 kmh^{-1} and he can walk through plot $BCDE$ at 8 kmh^{-1} . The total time T hours taken for him to reach D is given by the formula

$$T = \frac{1}{4}\sqrt{x^2 + 25} + \frac{1}{8}\sqrt{x^2 - 10x + 50}$$

- Show that, when $x = 3$, $T = 2.13$ to 2 decimal places.
- Find the values for T when $x = 0, 1, 2, 4, 5$.
- Choosing sensible scales, use your values from a and b to plot a graph of T against x on the graph paper opposite.
- Find the minimum value of T , stating the corresponding value of x .
- On a different day, Old McDonald walked in a straight line from A to E and then in a straight line from E to C . Find his average speed for his journey AEC .



TURN OVER

- 7 Study the pattern of numbers in the table below carefully.

	A	B	C	D
Row 0	0	0	1	1
Row 1	1	1	3	27
Row 2	4	5	5	125
Row 3	9	14	7	343
Row 4	16	30	9	729
Row 5				
Row 6				
Row n	a	b	c	d

- a Write down entries in columns A,B,C, and D for Row 5 and Row 6.
- b Explain how the entries in column B relate to the entries in column A.
- The entries in Row n are labelled a , b , c , and d .
- c Write down a formula for d in terms of c .
- d Given that $d = kb + c$ for some whole number k , which you should calculate, find a formula for b in terms of c .
- e Use your formula from part d to find the sum of the first 100 square numbers.

TURN OVER

END OF PAPER

Additional Answer Space

Additional Answer Space

Additional Answer Space

Additional Answer Space